Measuring the error of dynamic hedging: a Laplace transform approach

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Abstract

Using the Laplace transform approach, we compute expected value and variance of the error of a hedging strategy for a contingent claim when trading in discrete time. The method applies to a fairly general class of models, including Black-Scholes, Merton's jump-diffusion and Normal Inverse Gaussian, and to several interesting strategies, as the Black-Scholes delta, the Wilmott's improved-delta and the locally risk-minimizing strategy. The formulas obtained are valid for any fixed number of trading dates, whereas all previous results are asymptotic approximations. They can also be employed under model mispecification, to measure the influence of model risk on a hedging strategy.

1 Introduction

The object of this paper is the measurement of the hedging error due to trading in discrete time, usually referred to as "discretization error". Most of the financial models for pricing and hedging derivatives assume that trading is possible in continuous time. Of course, such an assumption does not hold for practical applications. For example, the widely used Black-Scholes delta hedging strategy produces a discretization error even if all other assumptions of the model are met. The discretization error depends on the path followed by the underlying asset until maturity and hence, even computing the variance of the error on a claim as simple as a European call, can be a very hard task. The practical importance of such a computation is self-evident, since it provides a way to measure the risk involved with discrete trading and, consequently, to quantify a compensation for it.

Quantifying the discretization error associated to a hedging strategy is a problem that is relevant both from a practical and a theoretical point of view and has been addressed by many papers in the literature. Hayashi and Mykland [12] use a weak convergence argument to derive the asymptotic distribution of the hedging error as the number of trades goes to infinity. Their approach was generalized by Tankov and Voltchkova [22] to processes with jumps. Some approximating formulas for the variance have been obtained, under the assumption of small trading intervals and for the log-normal model, by Kamal and Derman [13], by Mello and Neuhaus [16] and, in presence of transaction costs, by Toft [23]. In a slightly more general model setting, Gobet and Temam [9] proved asymptotic results about the L^2 norm of the error induced by the model-delta hedging for options with irregular payoffs. The fact that such approximations hold for vanishing time intervals constitutes an important limitation to their application, since in this case the error would also vanish.

A related, very important, problem is that of determining a strategy that minimizes the variance of the hedging error in an incomplete market. An extremely rich branch of the financial literature flourished after the seminal papers of Föllmer and Sondermann [7]. Schweizer [20] contains a review of the main results and contributions. The general solution in a discrete setting is given by Schweizer [21], who provided a characterization of the optimal strategy and a general formula for the optimal variance. However, an explicit computation for practical application is usually quite burdensome. For this reason, some algorithms useful for actual implementation have been proposed, for example by Bertsimas et al. [3] , with a dynamic programming approach or by Wilmott [24], with an independent approach, specific to the Black-Scholes model and based on second order approximation, who suggests a very easily implementable trading strategy. In the log-normal model, an exact computation of the minimal variance for any trading interval has been given by Angelini and Herzel [2], under the restriction that the underlying is a martingale.

A breakthrough in the problem of determining an efficient way to compute optimal strategies and their associated variances is proposed by Hubalek et al. [11] and by Černý [4]. Their idea is to consider contingent claims whose payoff function can be written as an inverse Laplace transform. They show that, under quite general assumption on the dynamics of the underlying, it is possible to compute the optimal strategy and its variance as an inverse Laplace transform of a function that depends on the claim and on the underlying process. This represents a relevant contribution from a practical point of view, since inverse Laplace transform can be evaluated very efficiently with standard numerical algorithms.

The present paper follows such approach, with the main objective of determining an efficient way to compute the first two moments of the distribution of the hedging error for "sub-optimal" strategies, such as the popular Black-Scholes delta. The key idea is to express the strategy itself as an inverse Laplace transform, so that one can directly compute the Laplace transform of the hedging error and, from it, its expected value and variance. The formulas obtained are valid for any fixed number of trading dates, whereas all previous results are asymptotic approximations. From a numerical point of view, we need to invert a Laplace transform in one or two dimensions. While there are several methodologies available for the one-dimensional case, the two-dimensional case is less explored. We implement an algorithm by Choudhury et al. [5]. In our experience we get accurate and fast results. The increasing computational power makes Monte Carlo approach a viable alternative for all computational problems, even when closed formulas are available. Closed formulas are mostly necessary when speed and accuracy play an important role. For instance, to compute the sensibility to different parameters by differentiating, which is often impractical with Monte Carlo, or to solve optimization problems. We will show a comparison of computational costs arising in the two cases as well as an application to an optimization problem.

We are able to assess the precision of the approximations, that hold under much more restrictive assumptions on the model and on the claim, like those of Kamal and Derman [13] and Toft [23]. Moreover, our result can be applied to measure the performance of a hedging strategy under model misspecification. For example, in the case of a trader that detects a market implied volatility higher than what she/he expects and wishes to exploit it. In this case, we can measure the expected performances of the hedging strategies in terms of Sharpe ratios. From a practical point of view this could help a trader to choose which trading strategy to adopt.

The rest of the paper is composed as follows: Section 2 contains the general setting and defines the class of strategies whose hedging error will be measured. Section 3 contains the main result. We give the details of the numerical implementation in Section 4, showing some applications in Section 5. Section 6 concludes and indicates some possible developments.

2 Model setting and integral representation

Let $(\Omega, \mathcal{F}, (\mathcal{F}_n)_{n\in(0,1,\dots,N)}, P)$ be a filtered probability space. We consider a one-dimensional process

$$S_n = S_0 \exp(X_n),$$

where the process $X = (X_n)$ for $n = 0, 1, \dots, N$, satisfies

1. X is adapted to the filtration $(\mathcal{F}_n)_{n\in(0,1,\dots,N)}$,
2. $X_0 = 0$,
3. the increments $\Delta X_n = X_n - X_{n-1}$ have the same distribution for $n = 1, \dots, N$,
4. ΔX_n is independent from $\mathcal{F}_{n-1}$ for $n = 1, \dots, N$.

We denote the moment generating function of X_1 by $m(z)$. We assume that $E[S_1^2] < \infty$ so that the moment generating function $m(z)$ is defined at least for complex z with $0 \le Re(z) \le 2$. Moreover, we exclude the case when S is a deterministic process. We suppose, without loss of generality, that the risk-free rate is zero or, equivalently, that S represents a discounted price.

Following the approach proposed by Hubalek et al. [11] we consider square-integrable, $\mathcal{F}_N$-measurable European contingent claims written on S with maturity T and payoff $H = f(S_N)$, where $f : (0, \infty) \to \mathbb{R}$ is of the form

$$f(s) = \int s^z \Pi(dz), \tag{2.1}$$

for some finite complex measure Π on a strip in the complex plane. Condition (2.1) states that the payoff function can be written as an inverse Laplace transform. For instance, for a European call option with strike price $K > 0$, the function $(s-K)^+$ may be written as

$$(s-K)^+ = \frac{1}{2\pi i}\int_{R-i\infty}^{R+i\infty} s^z \frac{K^{1-z}}{z(z-1)} dz,$$

for an arbitrary $R > 1$ and for each $s > 0$. For more details and other examples of integral representation of payoff functions we refer to [11].

Let $\vartheta = (\vartheta_n)$, for $n = 1, \dots, N$, be an admissible trading strategy with cumulative gains $G_n(\vartheta) = \sum_{k=1}^{n} \vartheta_k \Delta S_k$. An admissible strategy is a predictable process such that the cumulative gains are square-integrable (see [11] or [21]). Note that, because of the assumption of a null interest rate, the money market account does not contribute to the cumulative gain. The hedging error of the strategy is

$$\varepsilon(\vartheta, c) = H - c - G_N(\vartheta).$$

The random variable ϑ_n may be interpreted as the number of shares of the underlying asset held from time $n-1$ up to time n. If there exists a riskless asset, the strategy ϑ determines a unique self-financing portfolio and the hedging error $\varepsilon(\vartheta, c)$ may be viewed as the net loss one can suffer at maturity if one starts with the initial capital c and follows the strategy. We consider the problem of evaluating its expected value $E[\varepsilon(\vartheta, c)]$ and its variance $\mathrm{var}(\varepsilon(\vartheta, c))$.

It is well known that, for each initial endowment c, there exists a strategy which minimizes the expected square value of the hedging error

$$\text{Minimize } E\left[(H - c - G_N(\theta))^2\right] \text{ over all } \theta \in \Theta \tag{2.2}$$

(Schweizer [21]). We indicate with $\xi^{(c)}$ the solution of the problem and we call it the *variance-optimal strategy with given initial endowment.* Also one may consider the problem

$$\text{Minimize } E\left[(H - c - G_N(\theta))^2\right] \text{ over all } \theta \in \Theta \text{ and all } c \in \mathbb{R}. \tag{2.3}$$

The strategy ϕ and the initial endowment V_0 solutions of Problem (2.3) are called respectively the *variance-optimal strategy* and the *variance-optimal* *initial capital*. The optimal variance, namely the variance of the hedging error of ϕ, is effectively computed by Hubalek et al. [11] under the same assumptions on the process of the underlying as the present paper. In this context, it also plays an important role the *locally risk-minimizing strategy*, that is the strategy which minimizes the variance of the next period costs (for formal definition and properties, see [21]), which coincides with the variance-optimal strategy in the martingale case.

In practice, the most widely used trading strategy is still the Black-Scholes delta. It is therefore important to be able to compute the expected value and the variance of the related hedging error and to analyze the performance of delta hedging compared to that of variance-optimal strategies.

First of all we note that the Black-Scholes price at time n for claims satisfying Condition (2.1) can be computed as

$$C_n^{bs} = E_n^{bs}[H] = E_n^{bs}\left[\int S_N^z \Pi(dz)\right],$$

where E_n^{bs} is the Black-Scholes risk neutral expectation conditional to $\mathcal{F}_n$, which assumes i.i.d. and normal increments, with (annual) volatility σ. By Fubini's theorem, we can exchange the expected value with the integral in the complex variable to get

$$\begin{aligned} C_n^{bs} &= \int E_n^{bs}\left[S_N^z\right]\Pi(dz) = \int S_n^z E_n^{bs}\left[\exp(z(\Delta X_{n+1} + \ldots \Delta X_N))\right]\Pi(dz) \\ &= \int S_n^z m^{bs}(z)^{N-n}\Pi(dz), \end{aligned}$$

where

$$m^{bs}(z) = \exp\left(\left(-\frac{\sigma^2}{2}z + \frac{\sigma^2}{2}z^2\right)\frac{T}{N}\right).$$

Therefore, the units of underlying held from time $n-1$ to time n, when following the Black-Scholes delta hedging strategy, are

$$\Delta_n = \frac{\partial C_{n-1}^{bs}}{\partial S_{n-1}} = \int m^{bs}(z)^{N-n+1}\frac{\partial S_{n-1}^z}{\partial S_{n-1}}\Pi(dz) = \int z m^{bs}(z)^{N-n+1} S_{n-1}^{z-1}\Pi(dz)$$

Motivated by this computation, we consider admissible trading strategies ϑ which satisfy the following condition:

$$\vartheta_n = \int f^{\vartheta}(z)_n S_{n-1}^{z-1}\Pi(dz), \qquad (2.4)$$

for $n = 1, \ldots, N$, where $f^{\vartheta}(z)_n$ is a deterministic function of the complex variable z. Analogously to Condition (2.1), Condition (2.4) requires that ϑ_n depends only upon S_{n-1} and that it has a representation as an inverse Laplace transform.

In Condition (2.4) the measure Π is the same as that of Condition (2.1) for the contingent claim. This is not strictly needed for the approach presented in the paper to work. Different measures may arise when the strategy is not directly connected to the claim, as in the case of a stop-loss strategy. However, for the sake of a clearer exposition, we choose to connect each strategy to its associated contingent claim and use the same measure.

The Black-Scholes delta hedging is not a unique case; other interesting strategies which satisfy Condition (2.4) are:

- the locally risk-minimizing strategy. This is given in Theorem 2.1 in [11]:

$$\xi_n = \int f^{\xi}(z)_n S_{n-1}^{z-1} \Pi(dz),$$

  where

$$f^{\xi}(z)_n = g(z) h(z)^{N-n},$$

  with

$$\begin{aligned} g(z) &= \frac{m(z+1) - m(1)m(z)}{m(2) - m(1)^2}, \\ h(z) &= m(z) - (m(1) - 1)g(z). \end{aligned}$$

- the "improved-delta" strategy proposed by Wilmott [24] for the Black-Scholes model, that is an easily implementable approximation of the locally risk-minimizing strategy

$$\begin{aligned} &\Delta_n^w = \Delta_n + \frac{T}{N}(\mu - \frac{1}{2}\sigma^2)\Gamma_n S_{n-1} = \\ &= \int S_{n-1}^{z-1} \left(z m^{bs}(z)^{N-n+1} + \frac{T}{N}(\mu - \frac{1}{2}\sigma^2) z(z-1) m^{bs}(z)^{N-n+1} \right) \Pi(dz), \end{aligned}$$

  where μ and σ are respectively the drift and the volatility of the process. The expression of Δ_n^w can be obtained in an analogous way as for the delta, since the gamma of the claim Γ_n is the second derivative of C_{n-1}^{bs} with respect to S_{n-1}.

The delta and the improved-delta strategies are conceived for normal increments. Nevertheless, they may also be considered when the increments are not normal and our result below will still apply. In this case, a possible choice for parameters μ and σ would be to fit mean and variance of the distribution, or to take σ to minimize the variance of the hedging error. Notice that the common practice of using delta hedging at the current implied volatility does not satisfy Condition (2.4). This is also the case of "move based" strategies, where the position is adjusted only after a sufficiently ample movement of the price of the underlying. Other important examples which do not satisfy the condition are the optimal (in the sense of Problems (2.2) and (2.3)) strategies $\xi^{(c)}$ and ϕ.

3 Measuring the discretization error

For a contingent claim satisfying Condition (2.1), the hedging error of a strategy which satisfies Condition (2.4) has the following integral representation

$$\begin{aligned} \varepsilon(\vartheta, c) &= H - c - \sum_{k=1}^{N} \vartheta_k \Delta S_k \\ &= \int \left(S_N^z - \sum_{k=1}^{N} f^{\vartheta}(z)_k S_{k-1}^{z-1} \Delta S_k \right) \Pi(dz) - c, . \end{aligned} \tag{3.5}$$

The following theorem gives expected value and variance of the hedging error of such a strategy for a given initial capital c.

Theorem 3.1 *Let ϑ be an admissible strategy satisfying Condition (2.4) and let c be the initial value, then*

$$E[\varepsilon(\vartheta, c)] = \int S_0^z \left[m(z)^N - (m(1) - 1) \sum_{k=1}^{N} f^{\vartheta}(z)_k m(z)^{k-1} \right] \Pi(dz) - c \tag{3.6}$$

and

$$E[\varepsilon(\vartheta, 0)^2] = \int \int S_0^{y+z} (v_1(y, z) - v_2(y, z) - v_3(y, z) + v_4(y, z)) \Pi(dz) \Pi(dy), \tag{3.7}$$

where

$$v_1(y, z) = m(y + z)^N,$$

$$v_2(y,z) = \sum_{k=1}^{N} f^{\vartheta}(y)_k m(y+z)^{k-1} m(z)^{N-k}(m(z+1)-m(z)),$$

$$v_3(y,z) = \sum_{k=1}^{N} f^{\vartheta}(z)_k m(y+z)^{k-1} m(y)^{N-k}(m(y+1)-m(y)),$$

$$\begin{aligned} v_4(y,z) &= (m(2)-2m(1)+1)\sum_{k=1}^{N} f^{\vartheta}(y)_k f^{\vartheta}(z)_k m(z+y)^{k-1} + \\ &+ (m(1)-1)\sum_{k<j}\sum_{j=2}^{N} f^{\vartheta}(y)_k f^{\vartheta}(z)_j m(y)^{j-1-k} m(z+y)^{k-1}(m(y+1)-m(y)) + \\ &+ (m(1)-1)\sum_{j<k}\sum_{k=2}^{N} f^{\vartheta}(y)_k f^{\vartheta}(z)_j m(z)^{k-1-j} m(z+y)^{j-1}(m(z+1)-m(z)). \end{aligned}$$

Therefore, the variance of the hedging error is

$$\operatorname{var}(\varepsilon(\vartheta,c)) = \operatorname{var}(\varepsilon(\vartheta,0)) = E[\varepsilon(\vartheta,0)^2] - E[\varepsilon(\vartheta,0)]^2.$$

Proof. Since we assume that the hedging strategy and the contingent claim are square integrable, the hedging error is also square integrable and we can apply Fubini's Theorem. Given (3.5), we have

$$\begin{aligned} & E[H - \sum_{k=1}^{N} \vartheta_k \Delta S_k] = \int E[S_N^z - \sum_{k=1}^{N} f^{\vartheta}(z)_k S_{k-1}^{z-1}\Delta S_k]\Pi(dz) \\ = & \int \left\{ E[S_0^z \exp(z(\Delta X_1 + \ldots \Delta X_N))] - \sum_{k=1}^{N} f^{\vartheta}(z)_k E[S_{k-1}^{z-1}\Delta S_k] \right\} \Pi(dz) \\ = & \int S_0^z \left\{ m(z)^N - \sum_{k=1}^{N} f^{\vartheta}(z)_k E[\exp((z-1)(\Delta X_{k-1} + \ldots + \Delta X_1)) \right. \\ & \left. \times[\exp(\Delta X_k + \ldots + \Delta X_1) - \exp(\Delta X_{k-1} + \ldots + \Delta X_1)]] \right\} \Pi(dz) \\ = & \int S_0^z \left\{ m(z)^N - \sum_{k=1}^{N} f^{\vartheta}(z)_k \right. \\ & \left. \times E[(\exp(z(\Delta X_{k-1} + \ldots + \Delta X_1) + \Delta X_k) - \exp(z(\Delta X_{k-1} + \ldots + \Delta X_1))] \right\} \Pi(dz) \\ = & \int S_0^z \left[m(z)^N - \sum_{k=1}^{N} f^{\vartheta}(z)_k m(z)^{k-1}(m(1)-1) \right] \Pi(dz). \end{aligned}$$

which is (3.6). To prove (3.7) we need to compute

$$
\begin{aligned}
& E[(H-\sum_{k=1}^{N}\vartheta_k\Delta S_k)^2] \\
&= E[\int (H(z)-\sum_{k=1}^{N} f^{\vartheta}(z)_k S_{k-1}^{z-1}\Delta S_k)\Pi(dz)\int (H(y)-\sum_{k=1}^{N} f^{\vartheta}(y)_k S_{k-1}^{y-1}\Delta S_k)\Pi(dy)] \\
&= E[\int\int (H(z)-\sum_{k=1}^{N} f^{\vartheta}(z)_k S_{k-1}^{z-1}\Delta S_k)(H(y)-\sum_{k=1}^{N} f^{\vartheta}(y)_k S_{k-1}^{y-1}\Delta S_k)\Pi(dz)\Pi(dy)] \\
&= \int\int E[(H(z)-\sum_{k=1}^{N} f^{\vartheta}(z)_k S_{k-1}^{z-1}\Delta S_k)(H(y)-\sum_{k=1}^{N} f^{\vartheta}(y)_k S_{k-1}^{y-1}\Delta S_k)]\Pi(dz)\Pi(dy).
\end{aligned}
$$

Let us compute all the expectations needed:

$$
\begin{aligned}
E[H(z)H(y)] &= S_0^{y+z} E\left[\exp(z(\Delta X_N+\ldots+\Delta X_1)+y(\Delta X_N+\ldots+\Delta X_1))\right] \\
&= S_0^{y+z} m(y+z)^N = S_0^{y+z} v_1(y,z).
\end{aligned}
$$

$$
\begin{aligned}
& E[H(z)\sum_{k=1}^{N} f^{\vartheta}(y)_k S_{k-1}^{y-1}\Delta S_k] \\
&= \sum_{k=1}^{N} f^{\vartheta}(y)_k E[S_N^z S_{k-1}^{y-1}\Delta S_k] \\
&= \sum_{k=1}^{N} f^{\vartheta}(y)_k S_0^{z+y} E[\exp(z(\Delta X_N+\ldots+\Delta X_1))\exp((y-1)(\Delta X_{k-1}+\ldots+\Delta X_1)) \\
&\times [\exp(\Delta X_k+\ldots+\Delta X_1)-\exp(\Delta X_{k-1}+\ldots+\Delta X_1)]] \\
&= S_0^{z+y}\sum_{k=1}^{N} f^{\vartheta}(y)_k \\
&\times \{E[\exp((y+z)(\Delta X_{k-1}+\ldots+\Delta X_1))\exp(z(\Delta X_N+\ldots+\Delta X_k))\exp(\Delta X_k)] + \\
&- E[\exp((y+z)(\Delta X_{k-1}+\ldots+\Delta X_1))\exp(z(\Delta X_N+\ldots+\Delta X_k))]\} \\
&= S_0^{z+y}\sum_{k=1}^{N} f^{\vartheta}(y)_k \left[m(y+z)^{k-1}m(z)^{N-k}m(z+1)-m(y+z)^{k-1}m(z)^{N-k+1}\right] \\
&= S_0^{z+y} v_2(y,z).
\end{aligned}
$$

Analogously, computing the expectation

$$E[H(y)\sum_{k=1}^{N} f^{\vartheta}(z)_k S_{k-1}^{z-1}\Delta S_k]$$

one gets $S_0^{y+z}v_3(y,z)$.

The last term is

$$\begin{aligned}
&E[\sum_{k=1}^{N} f^{\vartheta}(z)_k S_{k-1}^{z-1}\Delta S_k \sum_{j=1}^{N} f^{\vartheta}(y)_j S_{j-1}^{y-1}\Delta S_j] \\
&= \sum_{k=1}^{N}\sum_{j=1}^{N} f^{\vartheta}(z)_k f^{\vartheta}(y)_j E[S_{k-1}^{z-1}S_{j-1}^{y-1}\Delta S_k \Delta S_j] \\
&= S_0^{y+z}\sum_{k=1}^{N}\sum_{j=1}^{N} f^{\vartheta}(z)_k f^{\vartheta}(y)_j \\
&\times E[\exp((z-1)(\Delta X_{k-1}+\ldots\Delta X_1))\exp((y-1)(\Delta X_{j-1}+\ldots\Delta X_1)) \\
&\times \exp(\Delta X_{k-1}+\ldots\Delta X_1)(\exp(\Delta X_k)-1) \\
&\times \exp(\Delta X_{j-1}+\ldots\Delta X_1)(\exp(\Delta X_j)-1)] \\
&= S_0^{y+z}\sum_{k=1}^{N}\sum_{j=1}^{N} f^{\vartheta}(z)_k f^{\vartheta}(y)_j \\
&\times E[\exp(z(\Delta X_{k-1}+\ldots\Delta X_1))\exp(y(\Delta X_{j-1}+\ldots\Delta X_1)) \\
&\times(\exp(\Delta X_k)-1)(\exp(\Delta X_j)-1)].
\end{aligned}$$

The last sum may be computed separating the cases $k=j$, $k<j$ and $k>j$ as

$$\begin{aligned}
&\sum_{k=j}\sum_{k=1}^{N} f^{\vartheta}(z)_k f^{\vartheta}(y)_k m(y+z)^{k-1}(m(2)-2m(1)+1)+ \\
&+\sum_{k<j}\sum_{j=2}^{N} f^{\vartheta}(z)_k f^{\vartheta}(y)_j m(y+z)^{k-1}m(y)^{j-1-k}(m(y+1)-m(y))(m(1)-1)+ \\
&+\sum_{k>j}\sum_{k=2}^{N} f^{\vartheta}(z)_k f^{\vartheta}(y)_j m(y+z)^{j-1}m(y)^{k-1-j}(m(z+1)-m(z))(m(1)-1),
\end{aligned}$$

which is $v_4(y,z)$. □

Theorem 3.1 states that the expected value and the variance of the hedging error may be represented respectively as one- and two-dimensional inverse Laplace transforms. Although the formulas look a bit involved, they can be easily evaluated numerically. In Section 4 we give some details on their implementation and discuss the precision of the algorithm used. A similar argument can be applied to compute higher order moments of the hedging error.

For Theorem 3.1 to hold it is not necessary that the strategy is consistent with the model. For instance, one can consider the case where the data generating process is the Black-Scholes process with a certain drift and volatility, while the strategy is based on different estimates. Or it may be the case that the data generating process is the Merton jump-diffusion process ([17]), while the strategy is conceived according to the Black-Scholes world, perhaps by fitting mean and variance of the increments.

A more general form of Theorem 3.1 holds if the increments ΔX_n are not identically distributed. All the computations would go through and one would get similar results by substituting all the powers of $m(\cdot)$ in the formulas with suitable products of the moment generating functions of each ΔX_n, for $n = 1, \dots, N$. Such a case may be useful to compute hedging errors on interest rate derivatives, since zero coupon bond prices have a non-constant volatility. Some recent results make us believe that it is possible to apply the method to the more general class of affine processes, which include many interesting cases such as the Heston stochastic volatility model.

As already noted, Condition (2.4) is fairly general, but excludes some interesting strategies, like the move-based (non-Markovian) ones, and those based on the implied volatility. For the computation presented, the most important requirements are the dependence only on the current price of the underlying and the representability as an inverse Laplace transform. The methodology might be extended to more general type of integrands depending also on some past prices, hence including some non-Markovian strategies.

Another important issue, not considered in this paper, is the presence of transaction costs. In this case, the cost of re-balancing the position adds to the cumulative gain of the strategy. Such an extra term depends on the strategy and may be any function of the mid-price process. It is possible to extend the present methodology to the case of proportional transaction costs when the hedging strategy satisfies a monotonicity property, that is when it buys (or sells) more units of the stock as its price increases. The problem is that the monotonicity condition is too restrictive and excludes many strategies, like, for instance, delta hedging.

Since a strategy satisfying Condition (2.4) does not depend on the initial endowment, the expected value of the error produced by such a strategy with initial value c can be obtained by simply subtracting c from the expected value of the same strategy with zero endowment. For the same reason, the variance of the error produced by such a strategy does not depend on c. On the other hand, the variance-optimal strategy with given initial capital $\xi^{(c)}$ does depend on the initial capital. The results in Hubalek et al. [11] allow to compute directly only the variance of the optimal strategy ϕ with optimal initial endowment. In the last section we compare performances, in terms of expected value and variance, of different strategies with fixed initial capital c, taking as a natural benchmark the variance-optimal strategy $\xi^{(c)}$. Hence we need to compute those quantities for the variance-optimal strategy with given initial endowment. The computation comes from general results in Schweizer [21].

Proposition 3.1 *Let the price process S_n satisfy assumptions of §2. Let ξ^c be the variance-optimal, N-step, strategy for a contingent claim H with an initial endowment c and let $\varepsilon(\xi^c, c)$ be its hedging error. Then*

$$E[\varepsilon(\xi^c, c)] = (V_0 - c)Q \tag{3.8}$$

where V_0 is the variance-optimal initial capital and

$$Q = \left(\frac{m(2) - m(1)^2}{m(2) - 2m(1) + 1}\right)^N .$$

Moreover, the variance of $\varepsilon(\xi^c, c)$ is

$$var(\varepsilon(\xi^c, c)) = var(\varepsilon(\phi, V_0)) + (V_0 - c)^2 Q(1 - Q), \tag{3.9}$$

where ϕ is the variance-optimal strategy (solution of Problem (2.3)).

Proof. From Corollary 2.5 in [21] it follows that

$$\begin{aligned} E[\varepsilon(\xi^c, c)] &= E[H\tilde{Z}^0] - cE[\tilde{Z}^0] \\ &= (V_0 - c)E[\tilde{Z}^0] \end{aligned}$$

where, using the fact that we are in the case of a deterministic mean-variance trade-off,

$$\tilde{Z}_0 = \prod_{k=1}^{N} (1 - \alpha_k \Delta S_k)$$

with

$$\alpha_k = \frac{E[\Delta S_k|\mathcal{F}_{k-1}]}{E[\Delta S_k^2|\mathcal{F}_{k-1}]}.$$

Setting

$$\lambda = \frac{(m(1)-1)}{m(2)-2m(1)+1},$$

we have

$$E[\tilde{Z}^0] = \prod_{k=1}^{N}(1-\lambda(m(1)-1)) = \left(\frac{m(2)-m(1)^2}{m(2)-2m(1)+1}\right)^N,$$

from which we get (3.8).

To see the statement on the variance, recall that, from Theorem 4.4 in [21],

$$\begin{aligned} &E[(H-c-G_N(\xi^{(c)})^2] = \mathrm{var}(H-G_N(\phi)) + \\ &+(V_0-c)^2\prod_{k=1}^{N}(1-\alpha_k E[\Delta S_k|\mathcal{F}_{k-1}]) = \\ &= \mathrm{var}(H-G_N(\phi)) + (V_0-c)^2\left(\frac{m(2)-m(1)^2}{m(2)-2m(1)+1}\right)^N. \end{aligned}$$

This gives (3.9).

□

4 Numerical implementation

There are at least three possible approaches to compute Formulas (3.6) and (3.7): numerical integration, fast Fourier transform and inversion of Laplace transform. We opt for the latter one. The formulas we wish to compute involve one- and two-dimensional Laplace transforms. The one-dimensional inversion, needed to compute expectations, is a standard technique for which several algorithms are available. A list of MATLAB codes for the one-dimensional case can be found in [14]. We adopted `invlap.m`, constructed by [10] and based on the method by [6], which is accurate and fast. The parameters of the algorithm in the code `invlap.m` for the one-dimensional inversion are two: the pole with largest real part of the function to be inverted and a tolerance parameter which essentially gives the distance of the vertical line of integration from the largest pole. To get a higher precision it is important to provide an exact value for the largest pole which depends on the claim to be hedged, on the strategy, and on the underlying model. For instance, in the standard case of delta hedging of a call option in the Black-Scholes setting, the largest pole is 1. The default value for the tolerance parameter (10^{-9}) gives, in our experience, rather accurate results.

The two-dimensional inversion is not as standard. We implement[1] the approach proposed by [5] and also used for a financial problem by Fusai and Tagliani [8]. It is basically an extension to two (or more) dimensions of the algorithm of Abate and Whitt [1] based on the Fourier-series method. The algorithm must be applied to functions defined on a finite domain. For this reason, the argument function is first damped by multiplying it by a decaying exponential function and then approximated by a periodic function. This produces the first source of error, the "aliasing error". The inversion formula is then obtained from the Fourier series of the periodic function. The series involved in the expansion have infinite terms: they are truncated and computed by using the Euler formula. This produces the second source of error, the "truncation error". The third source of error is the common "roundoff error" that in this case mainly comes from multiplications of large numbers by small ones. The algorithm has six parameters to control such errors: A_1, A_2 for the aliasing error, l_1,l_2 for the roundoff error, m and n for the truncation error. Each error may be decreased by increasing the corresponding parameters, with a consequent greater computational cost. For our applications we found that $A_1 = A_2 = 30$ and $l_1 = l_2 = 1$, suggested in [5], were good choices to get reasonably small aliasing and roundoff errors. We found that the parameters n and m, that control the Euler approximation to the infinite sums, are indeed the most relevant for our computations.

To give an idea on how the parameters n and m affect the precision and the computational cost of the algorithm, we choose a case where it is possible to compute the exact value of the square of the expected error. Let us consider an at-the-money European call option with maturity $T = 0.25$ years, hedged only once, at time 0, using the Black-Scholes delta. We assume that the underlying process is a Geometric Brownian motion with drift $\mu = 0.1$, volatility $\sigma = 0.4$ and that the initial price is $S_0 = 100$. In this case, the expected value and the variance of the hedging error produced by this static

[1] The code is available from the authors upon request.

strategy can be explicitly computed. The expected value, for an initial capital equal to the Black-Scholes price of the option, is 0.062723168. The one-dimensional algorithm produces an expected value equal to 0.062723143 with largest pole set to 1 and default tolerance parameter. The exact value of the expectation of the squared error is 103.5558. Table 1 reports the results produced by the algorithm as a function of m and n, keeping $A_1 = A_2 = 30$ and $l_1 = l_2 = 1$. The first column shows the difference (multiplied by 10^4) between the computed values and the exact ones. The second column reports the relative error. The number of terms in the approximating Euler sum computed by the algorithm is $n + m$, hence the computational burden increases with n and m. From this example (but it is our general impression), it appears that a higher accuracy is reached by increasing n, while larger m's provide just a marginal improvement. To give an idea of the computational costs we report (column 6), the cpu time (in seconds) necessary to carry on the operations on a desktop equipped with a dual processor at 2.13GHz.

Another possibility to get an estimate of the variance of the hedging error is the Monte Carlo method. In this case, like in many others, the main advantage of Monte Carlo is that it is easily implementable. On the negative side is the fact that, to achieve a given level of precision with a reasonable confidence, the computational burden may become unbearable. To evaluate the trade-off between advantages and disadvantages, we can compute the number of simulations necessary to achieve the same level of precision as the Laplace approach with a confidence level of 99%. This is reported in the fifth column of Table 1. The time necessary to perform 10^7 simulations on the same machine is 3.26 seconds. The trade-off between precision and speed is favorable for the Laplace transform and this may be particularly important in some applications. We show an example in Section 5.4.

5 Applications

5.1 Approximating formulas

Let us assess the precision of some approximations for the variance of the hedging error. Toft [23] provides a useful formula for computing an approximate value of the variance of the discretization error produced in the Black-Scholes model when hedging a European call option using the standard delta strategy. The formula, an approximation as the number of trading

m	n	abs.err$\cdot 10^4$	rel.err$\cdot 10^4$	Nsim	time (secs)
20	40	494.18	4.7721	$7.4 \cdot 10^7$	0.19
50	50	156.13	1.5077	$7.4 \cdot 10^8$	0.23
50	100	33.86	0.3270	$1.6 \cdot 10^{10}$	0.42
100	100	20.02	0.1933	$4.5 \cdot 10^{10}$	0.64
100	200	4.29	0.0414	$9.8 \cdot 10^{11}$	1.43
200	200	2.51	0.0242	$2.9 \cdot 10^{12}$	2.26
20	400	0.94	0.0091	$2.0 \cdot 10^{13}$	2.36
50	400	0.85	0.0082	$2.5 \cdot 10^{13}$	2.65
100	400	0.72	0.0069	$3.5 \cdot 10^{13}$	3.15
20	600	0.28	0.0027	$2.3 \cdot 10^{14}$	4.75

Table 1: Precision, computational cost and comparison to Monte Carlo of the two-dimensional inversion algorithm. The test is performed on the variance of the hedging error of the delta strategy for an at-the money call option with number of trading dates $N = 1$, maturity $T = 0.25$, initial value $S_0 = 100$, volatility $\sigma = 0.4$, drift $\mu = 0.1$. We compare different values of the parameters of the algorithm m and n. The other parameters are set to $A_1 = A_2 = 30$, $l_1 = l_2 = 1$. The exact value is $V = 103.5558$. The Table reports the values of the absolute errors $V(m, n) - V$, and the relative errors $(V(m, n) - V)/V$ (both multiplied by 10^4), the number $Nsim$ of Monte Carlo simulations needed to achieve the same precision with a 99% probability, the time (in seconds) employed by a dual processor at 2.13GHz to run the Laplace inversion algorithm. The time necessary to perform 10^7 simulations on the same machine is 3.26 seconds.

dates N goes to infinite, reads as follows

$$\begin{aligned} \operatorname{var}(\varepsilon(\Delta, c)) &\approx \frac{1}{2}\sigma^4 \left(\frac{T}{N}\right)^2 S_0^4 \Gamma_0^2 \sum_{i=0}^{N-1} g(t_i), \qquad (5.10) \\ g(t) &= \sqrt{\frac{T^2}{T^2 - t^2}} \exp\left(2\mu t - 2d_1 \frac{(\mu - r)t}{\sigma\sqrt{T}} - \frac{(\mu - r)^2 t^2}{\sigma^2 T}\right) \times \\ & \exp\left(\left[d_2^2 + 2d_2 \frac{(\mu - r)t}{\sigma\sqrt{T}} - \frac{(\mu - r)^2 t^2}{\sigma^2 T}\right] \frac{t}{T + t}\right), \end{aligned}$$

where Γ_0 is the option's gamma computed at time 0, and d_1 and d_2, the usual quantities in the Black-Scholes formula, are also computed at time 0.

A very popular approximation, proposed by Kamal an Derman [13], involving the option's vega κ_0 at time 0, is

$$\operatorname{var}(\varepsilon(\Delta, c)) \approx \frac{\pi}{4N}\sigma^2 \kappa_0^2. \qquad (5.11)$$

The top panel of Figure 1 represents the relative error of the two approximating formulas of standard deviation for number of trading dates $N = (1, 3, 5, 7, 10, 13, 26, 39, 52, 65)$. The parameters used are $S_0 = 100$, $r = 0$, $\mu = 0.05$, $\sigma = 0.5$, $T = 1$, $K = 100$. We see that, in this case, the approximation (5.10) underestimates the standard deviation while (5.11) overestimates it. The error of the first formula is above 4% when the trading intervals are fewer than 10 but goes under 2% as N gets greater than 26. Formula (5.11) gives similar, if not better results, especially for small values of N.

5.2 Comparing the strategies

We can compute the mean and the variance of the errors produced by different strategies to hedge a European call option. We suppose that the initial value of the strategy c is equal to the Black-Scholes value. To provide an ex-ante measurement of the performances of the strategies we compute expected values and standard deviations of their final shortfalls. Of course, the initial capital c only affects the expected values and not the standard deviations. To compute the standard deviation of the variance-optimal strategy $\xi^{(c)}$, which depends upon c, we calculate the optimal variance from [11] and then we use Proposition 3.1.

The natural goal for the hedger is to get a negative expected loss (i.e. a gain), hopefully with a small variance. A possible way to take both objectives into account is to compute the Sharpe index of the strategy

$$s(\vartheta, c) = \frac{-E[\varepsilon(\vartheta, c)]}{\sqrt{\mathrm{var}(\varepsilon(\vartheta, c))}}.$$

Let us compare the following strategies:

1. The Black-Scholes delta hedging strategy;
2. The "improved delta" strategy (Wilmott [24]);
3. The locally risk-minimizing strategy;
4. The variance-optimal strategy with initial capital c.

We consider an at-the-money European call option with maturity $T = 0.25$ years where the initial price of the underlying asset is $S_0 = 100$. We assume that a trader believes that the underlying follows a geometric Brownian motion with mean $\mu = 0.1451$ and volatility $\sigma = 0.4379$. The trader has the choice between different trading strategies and different numbers of trading dates, namely $N = (1, 3, 5, 7, 10, 13, 26, 39, 52, 65)$. We suppose that the price of the option is $c = 8.7176$, corresponding to the volatility σ. The trader sells the option, investing all the money in the hedging strategy.

We consider two cases for the data generating process of the underlying:

1. Geometric Brownian motion with parameters μ and σ (i.e. the trader is adopting the correct model)
2. Merton jump-diffusion process with normally distributed jumps, with parameters of the Geometric Brownian motion $\mu' = 0.05$ and $\sigma' = 0.3$, intensity of the jumps $\lambda = 10$, and mean and standard deviation of the jumps respectively $\nu = 0$ and $\tau = 0.1$. Note that, with such choice of parameters, the trader, although using an incorrect model, is estimating the correct values for the mean and the standard deviation of the returns.

We report that, for all instances analyzed, the improved-delta strategy is almost indistinguishable from the locally risk-minimizing one. Hence, we do not include it in the figures.

The results of the first case are reported in Figure 2. In the top panel we represent expected values of the total loss for the strategies considered as functions of the number of trading dates N. In the middle panel we plot the ratio between the standard deviation of each strategy and the minimal variance (achieved by the variance-optimal strategy). We note that, as the number of trading dates increases, the means and the variances of all the strategies go to zero, as expected since the model becomes complete in the limit. The expected value for the standard Black-Scholes delta is positive and quite different from those of the other strategies. The reason is that the positive μ is ignored by the delta hedging strategy but taken into account by all others. This is also reflected by the Sharpe indexes in the bottom panel of Figure 2 showing the worse ratio attained by the Delta hedging strategy.

In the second case we assume that the trader follows a strategy based on the Black-Scholes model, while the data generating process is the jump-diffusion Merton model. This is an application of Theorem 3.1 to the case of a strategy based on incorrect modeling assumptions. This type of analysis provides an insight on the influence of model risk on the performances of different hedging strategies. The trader may adopt the delta strategy or the locally risk-minimizing strategy, both based on the Black-Scholes model with the observed parameters. Notice that we cannot analyze the performance of the variance-optimal strategy based on a model other than the data generating one, because that would not satisfy Condition (2.4). However, the locally risk-minimizing Black-Scholes strategy is a good proxy for the variance-optimal one. Alternatively, if the trader had a perfect knowledge of the data generating process she/he could use either the locally risk-minimizing or the variance-optimal strategy. As usual, the variance-optimal strategy serves as a benchmark.

The results of the second case are represented in Figure 3. In this case, the model is not complete in the limit and therefore, neither the expected values (top panel) nor the standard deviations of the strategies go to zero. The smallest value (2.87) of the standard deviation is achieved for $N = 65$ by the variance-optimal strategy. Interestingly, the locally risk-minimizing Black-Scholes strategy performs better, with respect to the Sharpe index (bottom panel), than the standard delta strategy also in the case of model mispecification. In particular, the standard deviation of the delta strategy is consistently 2% higher than that of the variance-optimal one, while that of the log-normal locally risk-minimizing stays under 2%, as it is shown in Figure 3, middle panel.

5.3 Exploiting personal views

Now we assume that a trader has a view on the future values of the volatility of the underlying and wishes to implement a profitable strategy. In particular, we suppose that the market price of the option considered in the previous section is $c = 5.9785$, corresponding to an implied volatility $\sigma_0 = 0.3$. The trader believes that the underlying asset follows a Geometric Brownian motion with a lower volatility and therefore she/he sells the option and hedges it using the market implied volatility σ_0.

The results of this experiment are in Figure 4. The top panel reports the Sharpe indexes of the delta hedging and of the locally risk-minimizing strategy, assuming a drift rate $\mu_0 = 0.1$ and a number of trading dates $N = 10$, as the actual volatility σ ranges from 0.1 to 0.5. As expected, the Sharpe index is positive when σ is lower than σ_0, that is when the views of the trader are confirmed. We note that the locally risk-minimizing strategy gives a consistently slightly better performance. When $\sigma = \sigma_0$ the Sharpe index of the delta is negative (-0.0052), while that of the locally risk-minimizing strategy is positive (0.0099). In this case we can compute the Sharpe index of the variance-optimal strategy, which turns out to be 0.01. It is evident that the greater influence on the performance of the strategy is due to the difference between the hedging and the realized volatility, rather than to the strategy followed.

The lower panel of Figure 4 represents the influence of the drift on the Sharpe ratios of the two strategies. It appears that the Sharpe ratio of the locally risk-minimizing strategy is consistently better than that of the delta hedging when μ is not zero. In the martingale case the performances of the two strategies are very similar.

The analysis is concluded by Figure 5 that shows the influence of both the actual volatility σ and the actual drift μ on the Sharpe index of the locally risk-minimizing strategy, assuming a hedging volatility $\sigma_0 = 0.3$ and a hedging drift $\mu_0 = 0$. It is evident that the influence of the volatility is much stronger than that of the drift.

5.4 Optimal hedging volatility

Let us call *optimal hedging volatility* the volatility parameter that minimizes the variance of the error produced by the Black-Scholes delta hedging strategy, that is the optimal solution to the problem

$$\text{Minimize var}\left(H - \sum_{k=1}^{N} \delta_k(\sigma)\Delta S_k\right) \text{ over all } \sigma \in \mathbb{R},$$

where $\delta_k(\sigma)$ is the Black-Scholes delta at time $k-1$ as a function of the volatility parameter σ. Since we can compute the objective function of the optimization problem in closed form, we can determine efficiently the optimal value for σ. This is another advantage of closed formulas over approximation techniques such as Monte Carlo.

We consider three possibilities for the volatility parameter of the Black-Scholes delta strategy: the Black-Scholes implied volatility of the option, the volatility that fits the standard deviation of the increments and the optimal hedging volatility. We can compare the variances of the respective hedging errors, in particular with respect to the optimal variance. Notice that if the implied volatility is re-computed at each trading date, the resulting strategy does not satisfy Condition (2.4), hence the variance cannot be computed by our techniques. Hence, in this example we consider an implied volatility that is computed at time 0 and never re-adjusted.

We take as data generating process the Normal Inverse Gaussian (NIG) model with parameters $(\alpha, \beta, \delta, \mu)$. Its moment generating function is well known (see for instance [11]). We set μ so that the price process S is a martingale and hence the locally risk-minimizing strategy coincides with the variance-optimal one. We think to the parameters of the model as calibrated on a cross section of derivatives written on S. In other words, we assume that the probability measure under which the process S evolves is a martingale measure. We use three set of parameters corresponding respectively to values of the asymmetry of the distribution of increments -0.1482, 0, 0.1482. The parameters, expressed on annual basis, are respectively (16.0078, -2.5, 0.6325,0.0797), (14.9071, 0, 0.6112,-0.0205), (16.0078, 2.5, 0.6325, -0.1207). The three sets of parameters are such that the values of standard deviation and of excess kurtosis of the distribution are all the same, namely 20.25% and 0.3293 respectively.

We consider call options, with same maturity $T = 0.25$ and moneyness S/K from 0.8 to 1.2. The corresponding Black-Scholes implied volatilities are represented in Figure 6, left panels, for the different set of parameters, where the top graph corresponds to negative asymmetry, the middle one to zero asymmetry and the bottom one to positive asymmetry. The straight lines in the graphs represent the Black-Scholes volatility parameter that fits the standard deviation of the increments. We fix the number of trading dates to $N = 6$. We compute the optimal hedging volatility for each set of parameters and each moneyness. As shown in Figure 6, the optimal hedging volatility is usually greater than the other two, with the only exception of the positive asymmetry case in which, in the out-of-the-money region, the optimal hedging volatility and the implied volatility are very close. The right panels represent, as a function of moneyness and in the three asymmetry cases under exam, the ratio of the standard deviation of the hedging error of the three strategies with respect to that obtained with the variance-optimal strategy.

From this example it appears that choosing the optimal hedging volatility drastically reduces the variance of the final error, particularly for moneyness less than one when the asymmetry is negative and for moneyness greater than one when it is positive.

6 Conclusions

For a European contingent claim and an admissible strategy, both representable as inverse Laplace transforms, we are able to measure the error of the hedging in terms of expected value and variance.

The methodology applies to several interesting strategies, as the Black-Scholes delta, the improved-delta and the locally risk-minimizing one, and to a fairly general class of models, including Black-Scholes, Merton's jump-diffusion and Normal Inverse Gaussian. The formulas obtained are valid for any fixed number of trading dates.

Thanks to this method, it is possible to asses the precision of existing approximating formulas, as those proposed by [23] and [13] for the variance of the hedging error. One can also solve optimization problems such as that of determining, in a given underlying model, the volatility parameter that is optimal to use in a Black-Scholes delta hedging strategy. We compare the performance of different strategies under various model settings, taking as a benchmark the variance-optimal strategy and as a main performance measure the Sharpe index. The computations may be done also under model mispecification, providing a measure of the influence of model risk on hedging strategies.

The numerical analysis shows that a wrong choice of model parameters has a stronger impact on the hedging performances than the choice of the particular strategy adopted.

The method presented is limited to one-dimensional price processes with i.i.d. increments, in absence of transaction costs, and to strategies having a particular integral representation. It would be interesting to extend this approach to a more general setting.

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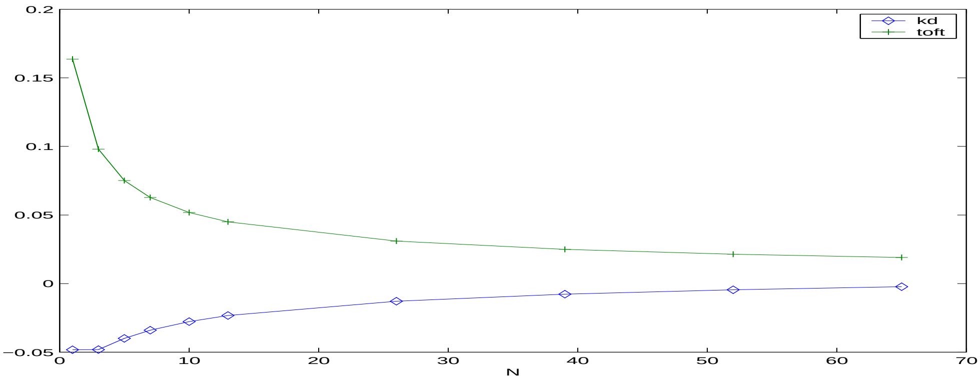


Figure 1 *Approximation errors of asymptotic formulas. Black-Scholes model with* $S_0 = 100$*,* $r = 0$*,* $\mu = 0.05$*,* $\sigma = 0.5$*, European call option with* $K = 100$*,* $T = 1$*. Relative error (1-approx/exact) of Toft's and Kamal-Derman's vega approximations of the standard deviation of the hedging error as a function of the number of trading intervals (*$N = (1, 3, 5, 7, 10, 13, 26, 39, 52, 65)$*).*

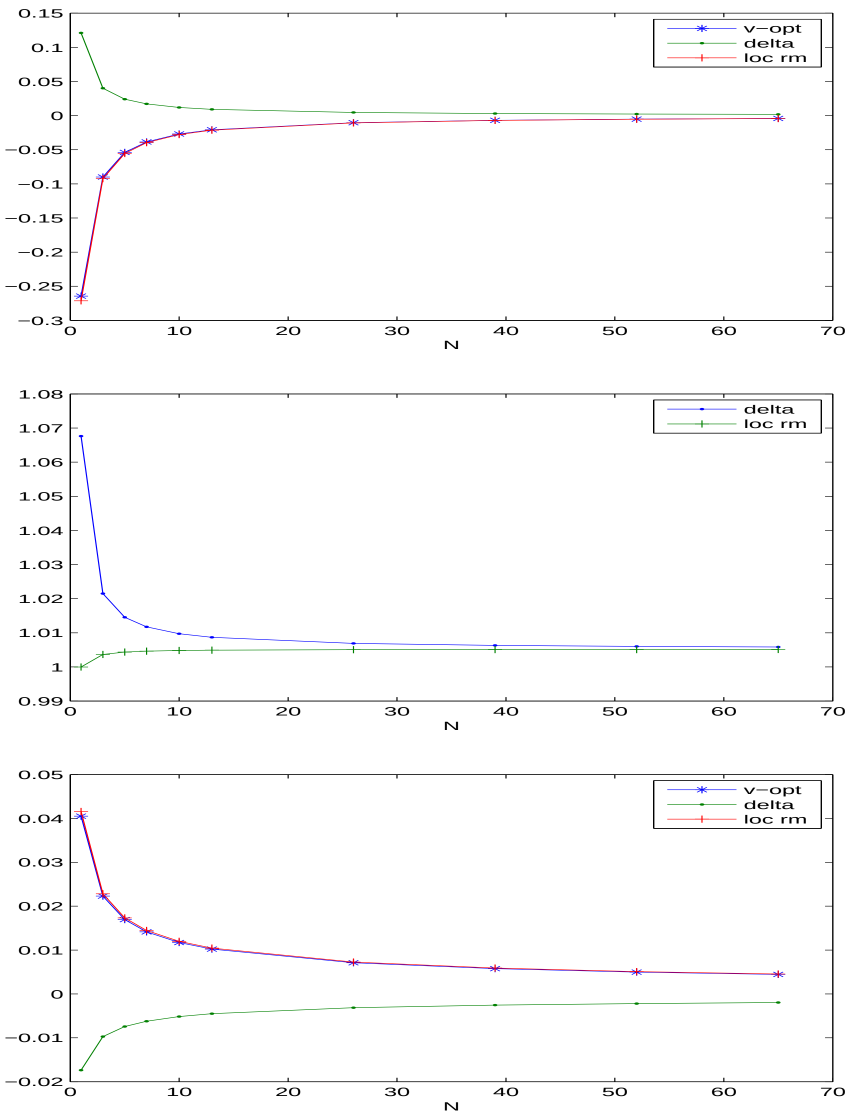
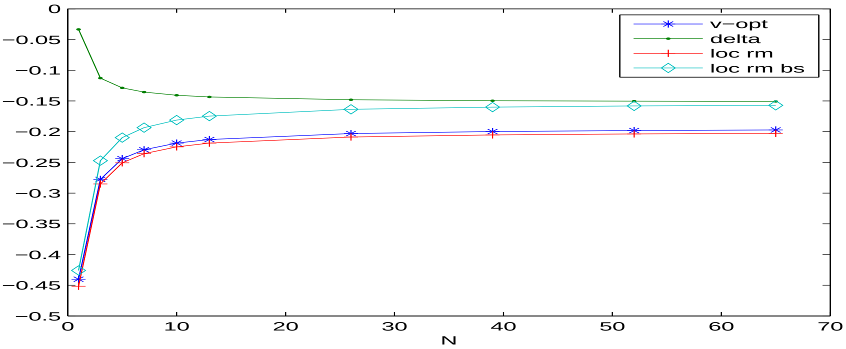
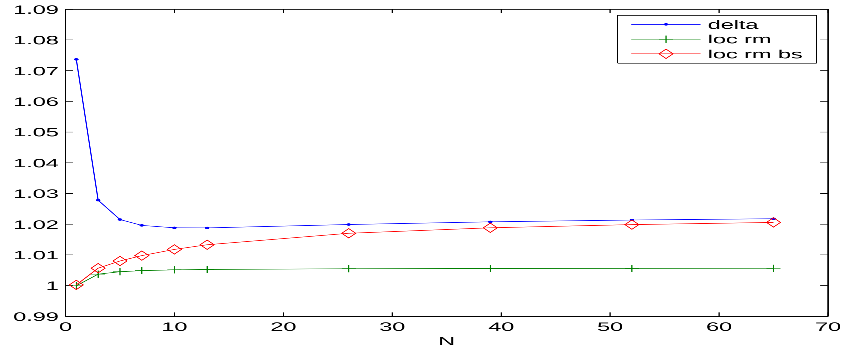


Figure 2 *Three measures of the hedging error of different strategies as the number of trading dates increases when the model (Black-Scholes) is correct. Expected value (top), ratio of standard deviation with respect to the variance-optimal one (middle) and Sharpe index (bottom) of hedging error.*





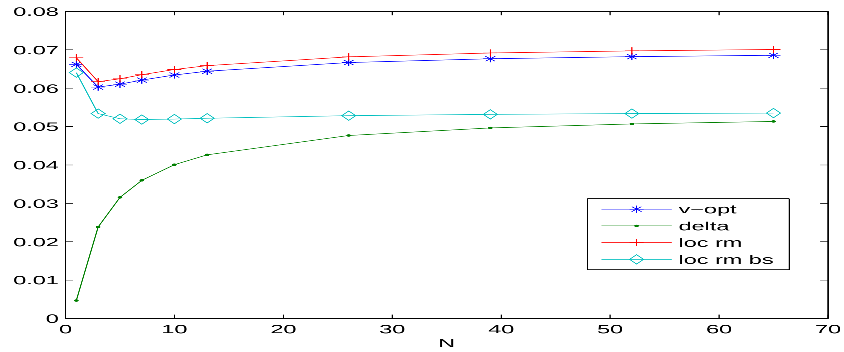


Figure 3 *Three measures of the hedging error of different strategies when the hedging model is incorrect. The delta and the locally risk-minimizing bs strategies are based on the Black-Scholes model, the locally risk-minimizing and the variance-optimal ones, are based on the data generating process that is Merton's jump-diffusion. The panels represent: expected value (top), ratio of standard deviation with respect to the variance-optimal one (middle) and Sharpe index (bottom) of hedging error of different strategies as the number of trading dates increases.*

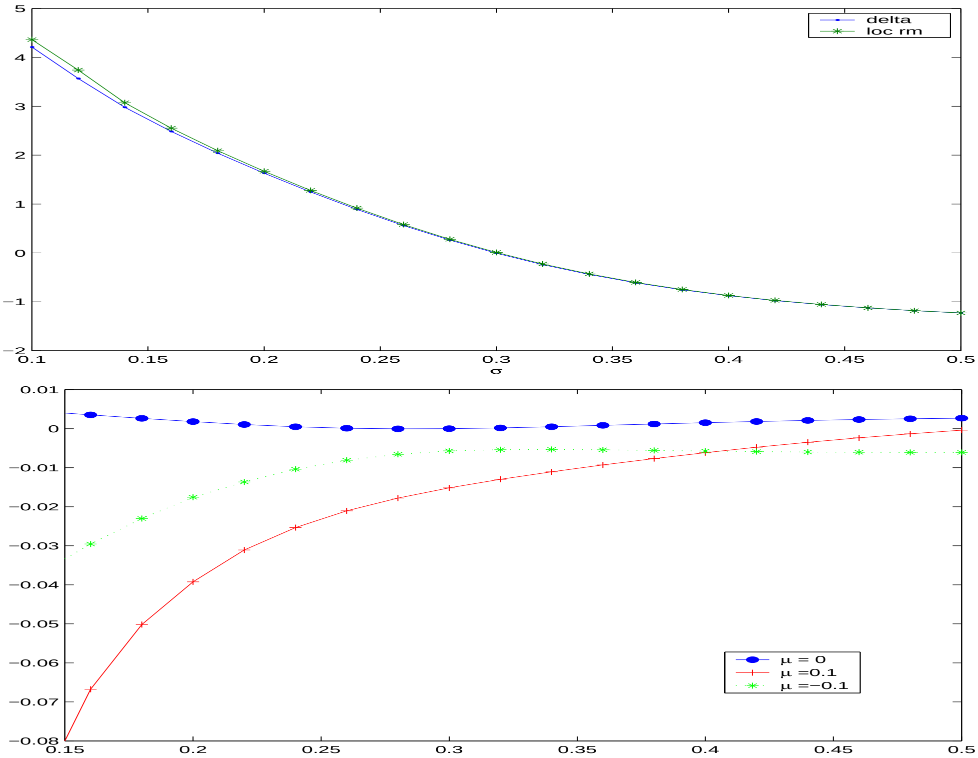


Figure 4 *Influence of the actual parameters on the performances of delta hedging and of the locally risk-minimizing strategies based on personal views on the volatility parameter* σ*. Black-Scholes model, with* $S_0 = K = 100$ *and number of trading dates* $N = 10$*. Top: Sharpe index of hedging error of delta and locally risk-minimizing strategies as a function of* σ*, the actual volatility of the Black-Scholes process. The strategies are constructed with* $\sigma_0 = 0.3$ *and* $\mu_0 = 0.1$ *(assuming that the value of* μ_0 *has been correctly estimated) . Bottom: differences between Sharpe indexes of hedging error of delta and of locally risk-minimizing strategies as a function of* σ *for different values of* μ*. The strategies are constructed with* $\sigma_0 = 0.3$ *and the correct value of* μ*. The three curves correspond to* $\mu = 0, 0.1, -0.1$*.*

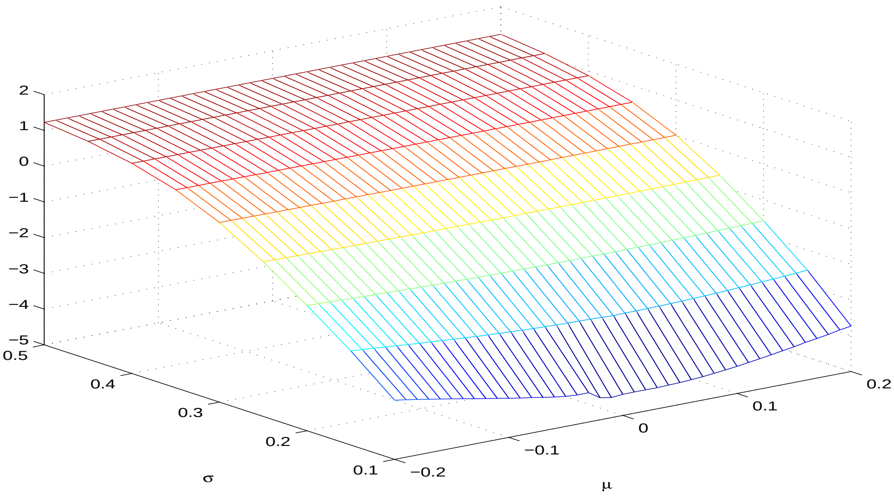


Figure 5 *Dependence of the Sharpe index of a locally risk-minimizing strategy on the actual parameters* μ *and* σ*. Black-Scholes model, with* $S_0 = K = 100$*, number of trading dates* $N = 10$*. The strategy is constructed assuming* $\sigma_0 = 0.3$ *and* $\mu_0 = 0$*.*

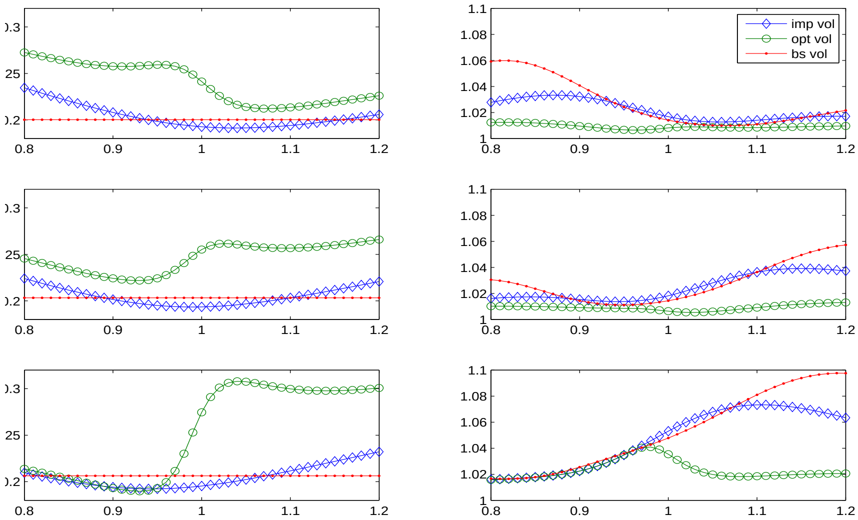


Figure 6 *Optimal hedging volatility versus implied volatility and volatility that fits the standard deviation of the increments as functions of the moneyness S/K (left panels). The data generating process is the NIG model with negative asymmetry (top), zero asymmetry (middle) and positive asymmetry (bottom). The left panels represent, as functions of the moneyness and in the three different asymmetry cases, the ratio between the standard deviation of the hedging error of the three strategies over that of the variance-optimal one.*